SOLID STATE PHYSICS 1

-March 2001-

I. Consider the honeycomb lattice drawn here, with one atom at each intersection point. Determine the following:

a) how many atoms are there in the primitive unit cell.

b) a primitive unit cell, the corresponding basis, and the primitive translation vectors $(\vec{a}_1, \vec{a}_2)$ in Cartesian coordinates.

Obs: Check again $(\vec{a}_1, \vec{a}_2)$ before proceeding.

c) the corresponding reciprocal lattice vectors $(\vec{b}_1, \vec{b}_2)$ in Cartesian coordinates.

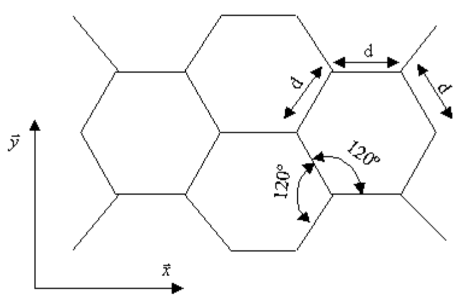
Hint: If you want to use the corresponding 3D (three-dimensional) formula, consider that this is only the top view of the crystal, and that the planes are separated by a distance *d*. This would lead to a simple form of $\vec{b}_3$ and to the same values of $(\vec{b}_1, \vec{b}_2)$ as in 2D case.

d) the Wigner-Seitz cell.

e) the following "crystal planes": (10), $(1\,\bar{1})$, (21), denoted in the system of translation vectors $(\vec{a}_1, \vec{a}_2)$ already chosen.

f) the structure factor $S(\vec{G}) = S(h,k)$ for the corresponding reciprocal vector $\vec{G} = h\vec{b}_1 + k\vec{b}_2$, assuming that all atoms have the same form factor f. Which diffraction spots have the strongest intensity?

Hint: Consider the case $h + k = 3m + r, \quad m \in Z, \quad r = 0,1,2$



Note: For drawing, you may use the drawings with the honeycomb lattice supplied at the end of the exam sheet.

II. Using the Debye approximation, show that in a *d*-dimensional harmonic crystal, the low-frequency density of normal modes varies as $^{d-1}$.

Hint: the volume of a sphere of radius *r* in *d* dimensions fulfills the relation: $V(r) \propto r^d$

III. a) Find the pressure exerted by a gas of electrons at 0 K, in terms of the density of electrons. Use the expression $p = -\left(\frac{\partial U}{\partial V}\right)_N$, where U is the total energy, V is the volume, and N is total number of particles.

b) Describe the distinction between a metal, a semiconductor, a semimetal and an insulator in terms of their energy band structure.

c) Why are usually current leads made of copper metal?

d) Why do you burn your finger easily when touching a hot metal?

e) Why do metals look shiny?

Obs: Answer questions c) , d) and e) not only by specifying the physical properties required, but also by giving arguments why they appear in metals.

IV. Consider the dispersion of the ferromagnetic magnons given by:

$$\hbar\omega = (2JSa^2)k^2$$

a) Calculate the heat capacity of the magnons in a 3D ferromagnet at low temperatures. Compare it with the contribution coming for the phonons $C_V^{Ph} \propto T^3$. Which one will dominate at very low temperatures?

b) Show that spin-wave theory predicts that a <u>two-dimensional</u> lattice will not be ferromagnetic at a finite temperature.

Hint: By using the dispersion relation, calculate the average number of spin waves at a finite temperature and show that it diverges.

Note: You can use the following results:

Plank's distribution formula for bosons: $n_k = \dfrac{1}{e^{\hbar\omega_k / kT} - 1}$

$$\int_0^\infty dx \frac{x^{3/2}}{e^x - 1} = 1.782 \qquad\qquad \frac{1}{e^x - 1} = \frac{d}{dx}\left[\ln(e^x - 1) - x\right]$$